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Filter-and-Forward Transparent Relay Design for OFDM Systems

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Abstract

In this paper, the filter-and-forward (FF) relay design for orthogonal frequency-division multiplexing (OFDM) transmission systems is considered to improve the system performance over simple amplify-and-forward (AF) relaying. Unlike conventional OFDM relays performing OFDM demodulation and remodulating, to reduce processing complexity, the proposed FF relay directly filters the incoming signal with a finite impulse response (FIR) and forwards the filtered signal to the destination. Two design criteria are considered to optimize the relay filter for given source power allocation. One is to minimize the relay transmit power subject to per-subcarrier signal-to-noise ratio (SNR) constraints and the other is to maximize the worst subcarrier channel SNR subject to the total relay transmit power constraint. It is shown that the two problems reduce to semi-definite programming (SDP) problems. Furthermore, the problem of joint source power allocation and relay filter design is considered for the second criterion, and a convergent iterative algorithm based on alternating optimization is proposed for the joint design. Numerical results show that the proposed FF relay significantly outperforms simple AF relays under both criteria. Thus, the proposed FF relay provides a practical alternative to the AF relaying scheme for OFDM transmission.

Keywords

Linear relay, filter-and-forward, amplify-and-forward, OFDM systems, semi-definite programming

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I. Introduction

Recently, relay networks have drawn extensive interest from the research community because they play an important role in enlarging the network coverage and improving the system performance in current and future wireless networks. Indeed, LTE-Advanced employs relays for coverage extension and performance improvement [1]. There are several well-known relaying schemes such as AF, decode-and-forward (DF), and compress-and-forward (CF) [2–4]. Among the relaying schemes, the AF scheme is the simplest and is suitable for cheap relay deployment under transparent* operation [1]. Recently, there have been some efforts to extend this simple AF scheme to a linear filtering relaying scheme, i.e., an FF scheme, to obtain better performance than the AF scheme while keeping the benefit of low computational complexity of the AF scheme [5–10]. It has been shown that the FF scheme can outperform the AF scheme considerably. However, most of the previous works on the FF relay have been done for single-carrier transmission, whereas most of the current wireless standards adopt OFDM transmission. Thus, in this paper, we propose direct FF relaying for OFDM transmission instead of using conventional OFDM relays which OFDM-demodulate the incoming signal, amplify or decode the demodulated signal, OFDM-remodulate the processed signal and transmit the remodulated OFDM signal to the destination [11–14]. In the proposed scheme, the incoming signal to the relay is FIR filtered at the *chip rate* of the OFDM modulations and the filtered signal is directly forwarded to the destination. In this way, the necessity of OFDM processing at the relay is eliminated, but the overall performance can still be improved over the AF scheme by a properly designed relay filter.

A. Our Approach and Contributions

There are several design criteria for the FF relay design for OFDM systems: rate maximization, minimizing power consumption, or overall quality-of-service (QoS) improvement. In this paper, we mainly consider power consumption minimization and QoS improvement under the scenario of single-input single-output (SISO) communication as the first step in this research direction.† First, we consider the case that the source power allocation is given and the FF relay filter is optimized. In this case, we consider two design criteria. One is to minimize the relay transmit power subject to per-subcarrier SNR constraints, and the other is to maximize the worst subcarrier channel SNR subject to a total relay transmit power constraint. The second criterion is particularly interesting from the perspective of overall QoS over subcarrier channels. Under the assumption of known channel state between the source and the relay and known channel statistic between

* Transparent operation means that the destination node does not know the existence of the relay node, and this operation is suitable for cheap AF relays [1].

† The multiple-input multiple-output (MIMO) communication case is beyond the scope of this paper and will be studied in a future work. For FF relay design for rate maximization, see Section V and the appendix.

the relay and the destination, which is suitable for cheap transparent relay operation, we show, by exploiting the eigen-property of circulant matrices and the structure of Toeplitz filtering matrices, that each problem can be reformulated as an SDP problem based on a semi-definite relaxation approach. Furthermore, for the first criterion, we show that the solution of the relaxed SDP problem is the same as that of the original problem under certain mild conditions. Next, we consider the case that both source power allocation and relay filter are optimized to maximize the worst subcarrier channel SNR under power constraints both at the source and the relay. For this joint optimization, we propose an iterative algorithm based on alternating optimization between linear programming for source power allocation and a SDP programming for the relay filter design, which converges to a local optimum at least. Numerical results show that the proposed FF relay significantly outperforms simple AF relays under both criteria. Thus, the proposed FF relay provides a practical alternative with low complexity to the AF relaying scheme for OFDM transmission.

B. Notation and Organization

In this paper, we will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a scalar a, a^* denotes its complex conjugate. For a matrix $\mathbf{A}$, $\mathbf{A}^*$, $\mathbf{A}^T$, $\mathbf{A}^H$, and $\text{tr}(\mathbf{A})$ indicate the complex conjugate, transpose, conjugate transpose, and trace of $\mathbf{A}$, respectively. $\mathbf{A} \succeq 0$ and $\mathbf{A} \succ 0$ mean that $\mathbf{A}$ is positive semi-definite and that $\mathbf{A}$ is strictly positive definite, respectively. For two matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B} \succeq 0$. $\mathbf{I}_n$ stands for the identity matrix of size n (the subscript is omitted when unnecessary), and $\mathbf{0}_{m \times n}$ denotes a $m \times n$ matrix of all zero elements. The notation $\texttt{Toeplitz}(\mathbf{f}^T, N)$ indicates a $N \times (N + L_f - 1)$ Toeplitz matrix with N rows and $[\mathbf{f}^T, 0, \cdots, 0]$ as its first row vector, where $\mathbf{f}^T$ is a row vector of size L_f, and $\text{diag}(d_1, \cdots, d_n)$ means a diagonal matrix with diagonal elements $d_1, \cdots, d_n$. The notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is complex circularly-symmetric Gaussian distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\mathbb{E}\{\cdot\}$ denotes the expectation. $j = \sqrt{-1}$.

The remainder of this paper is organized as follows. The system model is described in Section II. In Section III, the FF relay design problems are formulated and solved by using convex optimization theory. The performance of the proposed design methods is investigated in Section IV, followed by the conclusion and discussion in Section V.

II. System Model

We consider a full-duplex relay network composed of a source node (basestation), a relay node and a destination node (terminal station), as shown in Figures 1 and 2, where the source employs OFDM modulation with N subcarriers, and each link performs SISO communication. We consider the case that the direct link

between the source and the destination is weak. Thus, for simplicity, we assume that there is no direct link between the source and the destination and that both the source-to-relay (SR) link and the relay-to-destination (RD) link are inter-symbol interference (ISI) channels modelled as FIR filters. We assume that the relay is an FF relay, i.e., the relay performs FIR filtering on the incoming signal at the chip rate of the OFDM modulation and transmits the filtered output immediately to the destination. Thus, the FF relay can be regarded as an additional ISI channel between the source and the destination. We assume that the introduction of FIR filtering at the relay does not make the length of the overall FIR channel between the source and the destination larger than that of the OFDM cyclic prefix. (We will see later in Section IV that only a few taps for the relay FIR filter are enough to obtain most of the gain of the FF relay over the AF relay.) Since our focus of using the FF relay in this paper is the transparent relay operation, we assume that the SR channel state is known to the relay and that the RD channel state is unknown to the relay but the RD channel distribution is known to the relay.[‡] Such an assumption is reasonable for the transparent relay operation since the destination node does not know the existence of the relays; thus, the destination node does not feedback information to the relay node directly.

Specifically, at the source, the length N data vector of OFDM symbols is given by $\mathbf{s} := [s[0], s[1], \cdots, s[N-1]]^T$, where each data symbol is assumed to be a zero-mean independent complex Gaussian random variable, i.e., $s[k] \sim \mathcal{CN}(0, P_{s,k})$ for $k = 0, 1, \cdots, N-1$. The time-domain vector $\mathbf{x}_s$ after normalized inverse discrete Fourier transform (IDFT) at the source is given by

$$\underbrace{\begin{bmatrix} x_s[0] \\ x_s[1] \\ \vdots \\ x_s[N-1] \end{bmatrix}}_{=:\mathbf{x}_s} = \frac{1}{\sqrt{N}} \underbrace{\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega_N & \cdots & \omega_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_N^{N-1} & \cdots & \omega_N^{(N-1)^2} \end{bmatrix}}_{=:\mathbf{W}_N} \underbrace{\begin{bmatrix} s[0] \\ s[1] \\ \vdots \\ s[N-1] \end{bmatrix}}_{=\mathbf{s}}, \tag{1}$$

where $\omega_N = e^{j\frac{2\pi}{N}}$. Let $\mathbf{w}_{k-1}^T$ denote the k-th row of the normalized IDFT matrix $\mathbf{W}_N$ for $k = 1, \cdots, N$. Then, $\mathbf{x}_s$ can be written as $\mathbf{x}_s = [\mathbf{w}_0^T\mathbf{s}, \ \mathbf{w}_1^T\mathbf{s}, \ \cdots, \ \mathbf{w}_{N-1}^T\mathbf{s}]^T$ and the covariance matrix $\mathbf{\Sigma}_{\mathbf{x}_s}$ of $\mathbf{x}_s$ is given

[‡]The channel state information for the SR channel can be obtained at the relay by channel estimation based on possible training or a preamble signal embedded in the transmitted signal from the source. The channel statistic information for the RD link can be obtained based on the size of the relay cell or some initial measurement when the relay is deployed. It will be shown later that the proposed FF relay design is robust against channel statistic mismatch for the RD channel.

by

$$\mathbf{\Sigma}_{\mathbf{x}_s} = \mathbb{E}\{\mathbf{x}_s {\mathbf{x}_s}^H\} = \begin{bmatrix} \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} & \mathbf{p}_s^T \mathbf{w}_1^* & \cdots & \mathbf{p}_s^T \mathbf{w}_{N-1}^* \\ \mathbf{w}_1^T \mathbf{p}_s & \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} & \cdots & \mathbf{p}_s^T \mathbf{w}_{N-2}^* \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{w}_{N-1}^T \mathbf{p}_s & \mathbf{w}_{N-2}^T \mathbf{p}_s & \cdots & \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} \end{bmatrix}, \tag{2}$$

where $\mathbf{p}_s = [P_{s,0}, P_{s,1}, \cdots, P_{s,N-1}]^T$, since

$$\mathbb{E}\{\mathbf{w}_i^T \mathbf{s}\mathbf{s}^H \mathbf{w}_j^*\} = \begin{cases} \mathbf{w}_{i-j}^T \mathbf{p}_s & \text{if } i > j, \\ \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} & \text{if } i = j, \\ \mathbf{p}_s^T \mathbf{w}_{j-i}^* & \text{if } i < j. \end{cases} \tag{3}$$

The vector $\mathbf{x}_s$ is attached by a cyclic prefix with length L_{CP}, i.e.,

$$\tilde{x}_s[n] = \begin{cases} x_s[n], & n = 0, 1, \cdots, N-1, \\ x_s[N+n], & n = -1, -2, \cdots, -L_{CP}, \end{cases} \tag{4}$$

and the cyclic prefix attached sequence $\tilde{x}_s[n]$ is transmitted from the source to the relay through the SR channel. Then, the received baseband signal at the relay is given by

$$y_r[n] = \sum_{l=0}^{L_f-1} f_l \tilde{x}_s[n-l] + n_r[n], \tag{5}$$

where $\mathbf{f} = [f_0, f_1, \cdots, f_{L_f-1}]^T$ is the channel tap coefficient vector of the SR channel known to the relay, L_f is the length of the SR FIR channel, and $n_r[n]$ is the additive white Gaussian noise at the relay with $n_r[n] \sim \mathcal{CN}(0, \sigma_r^2)$. At the relay, the received signal $y_r[n]$ is *FIR filtered at the chip rate of the OFDM transmission* and then transmitted immediately to the destination. Thus, the output signal at the relay at (chip) time n is given by

$$y_t[n] = \sum_{l=0}^{L_r-1} r_l \, y_r[n-l], \tag{6}$$

where $\mathbf{r} = [r_0, r_1, \cdots, r_{L_r-1}]^T$ is the FIR filter coefficient vector at the relay and L_r is the order of the FIR filter. Note that, when $L_r = 1$, the FF relay simply reduces to the AF relay. However, when $L_r > 1$, the FF relay is an extension of the AF relay with some amount of digital processing. Finally, the signal transmitted by the relay goes through the RD FIR channel to the destination; thus, the received signal at the destination is given by

$$y_d[n] = \sum_{l=0}^{L_g-1} g_l \, y_t[n-l] + n_d[n], \tag{7}$$

where $\mathbf{g} = [g_0, g_1, \cdots, g_{L_g-1}]^T$ is the FIR channel tap coefficient vector for the RD channel, L_g is the order of the RD FIR channel, and $n_d[n]$ is zero-mean white Gaussian noise with variance σ_d^2 at the destination. Here, we assume that the channel tap coefficient g_l, $l = 0, 1, \cdots, L_g - 1$, is independent and identically distributed (i.i.d.) according to $g_l \overset{i.i.d.}{\sim} \mathcal{CN}(0, \sigma_g^2)$, i.e., each tap is independently Rayleigh faded, and that the realization $\{g_l,\ l = 0, 1, \cdots, L_g - 1\}$ is not known to the relay but its distribution is known to the relay. By stacking the output symbols at the relay and the received symbols at the destination, we have the following vectors for the transmitted signal at the relay and the cyclic prefix portion removed received signal vector at the destination, respectively:

$$\mathbf{y}_t = \mathbf{RF}\tilde{\mathbf{x}}_s + \mathbf{R}\mathbf{n}_r \quad \text{and} \quad \mathbf{y}_d = \mathbf{GRF}\tilde{\mathbf{x}}_s + \mathbf{GR}\mathbf{n}_r + \mathbf{n}_d, \tag{8}$$

where

$$\begin{aligned}
\mathbf{y}_d &= [y_d[N-1], y_d[N-2], \cdots, y_d[0]]^T, & (9)\\
\mathbf{y}_t &= [y_t[N-1], y_t[N-2], \cdots, y_t[0], y_t[-1], \cdots, y_t[-L_g+1]]^T, & (10)\\
\tilde{\mathbf{x}}_s &= [x_s[N-1], x_s[N-2], \cdots, x_s[0], x_s[-1], \cdots, x_s[-L_g-L_r-L_f+3]]^T, & (11)\\
\mathbf{n}_r &= [n_r[N-1], n_r[N-2], \cdots, n_r[0], n_r[-1] \cdots, n_r[-L_g-L_r+2]]^T, & (12)\\
\mathbf{n}_d &= [n_d[N-1],\ n_d[N-2], \cdots, n_d[0]]^T, & (13)\\
\mathbf{G} &= \mathtt{Toeplitz}(\mathbf{g}^T, N), & (14)\\
\mathbf{R} &= \mathtt{Toeplitz}(\mathbf{r}^T, N+L_g-1), & (15)\\
\mathbf{F} &= \mathtt{Toeplitz}(\mathbf{f}^T, N+L_g+L_r-2). & (16)
\end{aligned}$$

Under the assumption that $L_{CP} \geq L_g + L_r + L_f - 3$, the DFT of the cyclic prefix portion removed received vector of size N at the destination is given by

$$\begin{aligned}
\hat{\mathbf{y}}_d &= \mathbf{W}_N^H\mathbf{GRF}\tilde{\mathbf{x}}_s + \mathbf{W}_N^H\mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H\mathbf{n}_d, & (17)\\
&= \mathbf{W}_N^H\mathbf{H}_c\mathbf{W}_N\mathbf{s} + \mathbf{W}_N^H\mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H\mathbf{n}_d, & (18)\\
&= \mathbf{Ds} + \mathbf{W}_N^H\mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H\mathbf{n}_d, & (19)
\end{aligned}$$

where $\mathbf{W}_N^H$ is the normalized DFT matrix of size N, $\mathbf{H}_c$ is a $N \times N$ circulant matrix generated from the overall Toeplitz filtering matrix $\mathbf{GRF}$ from the source to the destination, and $\mathbf{D} = \text{diag}(d_0, \cdots, d_{N-1}) = \mathbf{W}_N^H\mathbf{H}_c\mathbf{W}_N$ is the eigen-decomposition of $\mathbf{H}_c$.

A. Manipulation for quadratic forms

The received signal form (19) is standard in OFDM transmission, but the form cannot be used directly for relay filter optimization in the next section. Thus, in this subsection, we derive an explicit expression for the received signal $\hat{y}_d[k]$, $k = 0, 1, \cdots, N-1$, which facilitates optimization formulation in the next section, based on the following property of circulant matrices [15].

Lemma 1: [15] Let $\mathbf{C}$ be an $N \times N$ circulant matrix with the first row $[c(0), c(1), \cdots, c(N-1)]$. Then, the eigenvalues of $\mathbf{C}$ are given by

$$\lambda_k = \sum_{n=0}^{N-1} c(n)\omega_N^{-kn}, \quad k = 0, 1, \cdots, N-1,$$

with the corresponding right eigenvectors

$$\boldsymbol{\xi}_k = \frac{1}{\sqrt{N}}[1, \omega_N^{-k}, \omega_N^{-2k}, \cdots, \omega_N^{-(N-1)k}]^T, \quad k = 0, 1, \cdots, N-1.$$

By Lemma 1, to derive the diagonal elements of $\mathbf{D}$ in (19), we need to know only the first row of $\mathbf{H}_c$ in (18). Let the first row of $\mathbf{G}$ be $\tilde{\mathbf{g}}^T$. Then, $\tilde{\mathbf{g}}^T$ is a $1 \times (N + L_g - 1)$ row vector given by

$$\tilde{\mathbf{g}}^T = [\mathbf{g}^T, 0, \cdots, 0], \tag{20}$$

and the first row of $\mathbf{GRF}$ is given by $\tilde{\mathbf{g}}^T\mathbf{RF}$. Since $\mathbf{H}_c$ is generated by truncating out the elements of $\mathbf{GRF}$ outside the first $N \times N$ positions and by moving the lower $(L_g + L_r + L_f - 3) \times (L_g + L_r + L_f - 3)$ elements of the truncated part to the lower left of the untruncated $N \times N$ matrix, the first row $\mathbf{h}_c^T$ of $\mathbf{H}_c$ is simply the first N elements of the first row $\tilde{\mathbf{g}}^T\mathbf{RF}$ of $\mathbf{GRF}$, i.e., $\mathbf{h}_c^T = \tilde{\mathbf{g}}^T\mathbf{RFT}$, where $\mathbf{T}$ is a truncation matrix for truncating out the elements of $\tilde{\mathbf{g}}^T\mathbf{RF}$ except the first N elements, given by

$$\mathbf{T} = \begin{bmatrix} \mathrm{I}_N \\ \mathbf{0}_{(L_g+L_r+L_f-3)\times N} \end{bmatrix}. \tag{21}$$

Now, the diagonal elements of $\mathbf{D}$ can be obtained by Lemma 1 and are given by

$$[d_0, \cdots, d_{N-1}]^T = \sqrt{N}\mathbf{W}_N^H(\tilde{\mathbf{g}}^T\mathbf{RFT})^T, \tag{22}$$

where $\sqrt{N}\mathbf{W}_N^H$ is the DFT matrix of size N. Finally, the received signal in the k-th subcarrier at the destination is expressed as

$$\hat{y}_d[k] = \sqrt{N}\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}s[k] + \mathbf{w}_k^H\mathbf{GRn}_r + \mathbf{w}_k^H\mathbf{n}_d, \tag{23}$$

where $\mathbf{w}_k^H$ is the $(k+1)$-th row of $\mathbf{W}_N^H$. Thus, the signal and noise parts of $\hat{y}_d[k]$ are given by

$$\hat{y}_{d,S}[k] = \sqrt{N}\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}s[k] \quad \text{and} \quad \hat{y}_{d,N}[k] = \mathbf{w}_k^H\mathbf{GRn}_r + \mathbf{w}_k^H\mathbf{n}_d, \tag{24}$$

respectively, for $k = 0, 1, \cdots, N-1$.

III. Filter-and-Forward Relay Design Criteria and Optimization

In this section, we consider several meaningful FF relay design problems for the relay network employing OFDM transmission described in Section II. First, we consider the case that the source power allocation $\{P_{s,k}\}$ is given, and the FF relay filter is optimized. In this case, we consider two design criteria. The first criterion is to minimize the transmit power of the FF relay subject to a SNR constraint for each OFDM subcarrier channel, and the second criterion is to maximize the worst subcarrier SNR subject to a relay power constraint to improve overall QoS for subcarrier channels. Here, we shall show that each problem can be reformulated as a semi-definite programming (SDP) problem based on a semi-definite relaxation (SDP) approach. That is, the original non-convex FF relay design problems are approximated by convex SDP problems. For the first criterion, we shall show that the solution of the relaxed SDP problem is the same as that of the original problem under certain mild conditions. Next, we consider the case that both source power allocation and relay filter are optimized to maximize the worst subcarrier SNR under power constraints both at the source and the relay. For this joint optimization, we propose an iterative algorithm based on alternating optimization between linear programming for source power allocation and SDP programming for the relay filter design, which converges to a local optimum at least.

A. FF relay optimization for given source power allocation

A.1 FF relay transmit power minimization under per-subcarrier SNR constraints

We first consider the problem of designing the FF relay tap coefficient vector $\mathbf{r} = [r_0, r_1, \cdots, r_{L_r-1}]^T$ to minimize the relay transmit power subject to an SNR constraint per OFDM subcarrier. This problem is formulated as follows:

Problem 1: For given source power allocation $\{P_{s,k}, k = 0, 1, \cdots, N-1\}$, SR channel $\mathbf{f}$, RD channel static information (σ_g^2, L_g), FF relay filter order L_r, and a set $\{\gamma_k,\ k \in \mathcal{I}\}$ of minimum desired SNR values for subcarrier channels $\mathcal{I}$,

$$\min_{\mathbf{r}} \ P_r \qquad \text{subject to (s.t.)} \quad \text{SNR}_k \geq \gamma_k, \quad \forall \ k \in \mathcal{I} \subset \{0, 1, \cdots, N-1\} \tag{25}$$

where P_r is the relay transmit power and SNR_k is the SNR of the k-th subcarrier channel.

To solve Problem 1, we need to express each term in the problem as a function of the design parameter $\mathbf{r}$. First, let us derive the SNR on the k-th subcarrier channel in the received signal (23) at the destination. Note that the signal and noise parts in (24) are represented in terms of the relay filtering matrix $\mathbf{R}$. The representation of SNR in terms of $\mathbf{R}$ is redundant since $\mathbf{r}$ is embedded in $\mathbf{R}$. Thus, we need reparameterization of SNR in terms of $\mathbf{r}$, and this is done based on (24) by exploiting the Toeplitz structure of $\mathbf{R}$ as follows. Using (24), we

first express the received signal power at the destination in terms of $\mathbf{r}$ as

$$\begin{aligned}
\mathbb{E}\{|\hat{y}_{d,S}[k]|^2\} &= N\mathbb{E}\{\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}|s[k]|^2\tilde{\mathbf{g}}^H\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\},\\
&= N\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\mathbb{E}\{|s[k]|^2\tilde{\mathbf{g}}\tilde{\mathbf{g}}^H\}\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k,\\
&\overset{(a)}{=} NP_{s,k}\mathrm{tr}(\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\mathbb{E}\{\tilde{\mathbf{g}}\tilde{\mathbf{g}}^H\}\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k),\\
&\overset{(b)}{=} NP_{s,k}\sigma_g^2\mathrm{tr}(\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{I}}_{L_g}\tilde{\mathbf{I}}_{L_g}\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k),\\
&\overset{(c)}{=} NP_{s,k}\sigma_g^2\mathrm{tr}(\underbrace{\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T}_{=:\mathbf{K}_k}\mathbf{R}_{L_g}^T\mathbf{R}_{L_g}^*),\\
&\overset{(d)}{=} NP_{s,k}\sigma_g^2\mathrm{tr}(\mathbf{R}_{L_g}^*\mathbf{K}_k\mathbf{R}_{L_g}^T),\\
&\overset{(e)}{=} NP_{s,k}\sigma_g^2\left[\mathrm{vec}(\mathbf{R}_{L_g}^T)\right]^H\bar{\mathbf{K}}_k\mathrm{vec}(\mathbf{R}_{L_g}^T)\\
&= NP_{s,k}\sigma_g^2\left[\mathrm{vec}(\mathbf{R}^T)\right]^H\ \tilde{\mathbf{K}}_k\ \mathrm{vec}(\mathbf{R}^T)\\
&\overset{(f)}{=} NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}, \qquad (26)
\end{aligned}$$

where

$$\tilde{\mathbf{I}}_{L_g} := \begin{bmatrix} \mathbf{I}_{L_g} & \mathbf{0}_{L_g\times(N-1)} \\ \mathbf{0}_{(N-1)\times L_g} & \mathbf{0}_{(N-1)\times(N-1)} \end{bmatrix};\quad \mathbf{R} = \begin{bmatrix} \mathbf{R}_{L_g} \\ \mathbf{R}_{N-1} \end{bmatrix};\quad \bar{\mathbf{K}}_k = \mathbf{I}_{L_g}\otimes\mathbf{K}_k;\quad \tilde{\mathbf{K}}_k = \tilde{\mathbf{I}}_{L_g}\otimes\mathbf{K}_k;$$

$\mathbf{R}_{L_g} = \tilde{\mathbf{I}}_{L_g}\mathbf{R}$ is a matrix composed of the first L_g rows of $\mathbf{R}$; and

$$\mathbf{E}_1 = [\ \underbrace{\mathbf{I}_{L_r},\mathbf{0}_{L_r\times(N+L_g-2)}}_{N+L_g+L_r-2\ \text{ columns}},\underbrace{\mathbf{0}_{L_r\times 1},\mathbf{I}_{L_r},\mathbf{0}_{L_r\times(N+L_g-3)}}_{N+L_g+L_r-2\ \text{ columns}},\cdots,\ \underbrace{\mathbf{0}_{L_r\times(N+L_g-2)},\mathbf{I}_{L_r}}_{N+L_g+L_r-2\ \text{ columns}}\ ]. \qquad (27)$$

Here, (a) is due to the assumption of independence of the signal and the RD channel coefficients; (b) is due to the assumption[§] of $g_l \overset{i.i.d.}{\sim} \mathcal{CN}(0,\sigma_g^2)$ (see (20)) ; (c) and (d) are due to tr$(\mathbf{ABC})=$ tr$(\mathbf{CAB})$; (e) is due to tr$(\mathbf{R}_{L_g}^*\mathbf{K}_k\mathbf{R}_{L_g}^T) = \left[\mathrm{vec}(\mathbf{R}_{L_g}^T)\right]^H\bar{\mathbf{K}}_k\mathrm{vec}(\mathbf{R}_{L_g}^T)$; and (f) is obtained because $\mathbf{R} = \mathtt{Toeplitz}(\mathbf{r}^T, N+L_g-1)$ and thus vec$(\mathbf{R}^T) = \mathbf{E}_1^H\mathbf{r}$. The key point of the derivation of (26) is that the received signal power at the k-th subcarrier channel is represented as a quadratic form of the design variable $\mathbf{r}$. Next, consider the received noise power for the k-th subcarrier channel. Using similar techniques to those used to obtain (26), we can

[§]The i.i.d. assumption for g_l is not necessary and the assumption of independence is enough. Under the independence assumption, however, the formulation becomes more complicated.

express the received noise power based on the noise part in (24) as

$$\begin{aligned}
\mathbb{E}\{|\hat{y}_{d,N}[k]|^2\} &= \sigma_r^2 \text{tr}(\mathbf{R}^H \underbrace{\mathbf{G}^H \mathbf{w}_k \mathbf{w}_k^H \mathbf{G}}_{=:\mathbf{M}_k} \mathbf{R}) + \sigma_d^2, \\
&= \sigma_r^2 \left[\text{vec}(\mathbf{R})\right]^H \tilde{\mathbf{M}}_k \text{vec}(\mathbf{R}) + \sigma_d^2, \\
&= \sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2, \qquad (28)
\end{aligned}$$

where $\tilde{\mathbf{M}}_k = \mathbf{I}_{N+L_g+L_r-2} \otimes \mathbf{M}_k$ and $\mathbf{E}_2$ is given by

$$\mathbf{E}_2 = \begin{bmatrix}
\mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T & \mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T \\
\mathbf{0}^T & \mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\
\mathbf{0}^T & \mathbf{0}_z & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & & \ddots & \ddots & \ddots & \ddots & \mathbf{0}^T \\
\mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T & \mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T
\end{bmatrix}. \qquad (29)$$

Here, $\mathbf{e}_i^T$ be the i-th row of $\mathbf{I}_{N+L_g-1}$ and the size of each $\mathbf{0}^T$ in (29) is $1 \times (N + L_g - 1)$. (It is easy to verify that vec$(\mathbf{R}) = \mathbf{E}_2^H \mathbf{r}$ due to the Toeplitz structure of $\mathbf{R}$.) Based on (26) and (28), the SNR of the k-th subcarrier channel is now expressed as

$$\text{SNR}_k = \frac{N P_{s,k} \sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2}. \qquad (30)$$

Finally, consider the relay transmit power. Using (8), we obtain the relay transmit power in a similar way as

$$\begin{aligned}
\mathbb{E}\{\text{tr}(\mathbf{y}_t \mathbf{y}_t^H)\} &= \text{tr}(\mathbf{R}\mathbf{F} \underbrace{\mathbb{E}\{\tilde{\mathbf{x}}_s \tilde{\mathbf{x}}_s^H\}}_{=:\mathbf{\Sigma}_{\tilde{\mathbf{x}}_s}} \mathbf{F}^H \mathbf{R}^H) + \text{tr}(\sigma_r^2 \mathbf{R}\mathbf{R}^H), \\
&= \text{tr}(\mathbf{R} \underbrace{(\mathbf{F}\mathbf{\Sigma}_{\tilde{\mathbf{x}}_s} \mathbf{F}^H + \sigma_r^2 \mathbf{I})}_{=:\mathbf{\Pi}} \mathbf{R}^H), \\
&= \left[\text{vec}(\mathbf{R}^H)\right]^H \tilde{\mathbf{\Pi}} \text{vec}(\mathbf{R}^H), \\
&= \mathbf{r}^T \mathbf{E}_1 \tilde{\mathbf{\Pi}} \mathbf{E}_1^H \mathbf{r}^* = \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r}, \qquad (31)
\end{aligned}$$

where $\tilde{\mathbf{\Pi}} = \mathbf{I}_{N+L_g-1} \otimes \mathbf{\Pi}$, and $\mathbf{\Sigma}_{\tilde{\mathbf{x}}_s}$ is obtained similarly to (2) based on (3). The last equality holds since the power is a real-valued quantity.

Now, based on (26), (28) and (31), Problem 1 can be restated as follows:

Problem 2: For the given parameters,

$$\min_{\mathbf{r}} \quad \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r} \quad \text{s.t.} \quad \frac{N P_{s,k} \sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2} \geq \gamma_k \;, \quad k \in \mathcal{I}. \qquad (32)$$

In general, Problem 2 is not a convex problem. However, the problem can still be solved efficiently by using convex optimization techniques. Let $\mathcal{R} := \mathbf{r}\mathbf{r}^H$. Then, by using tr$(\mathbf{ABC}) =$ tr$(\mathbf{BCA})$ and relaxing the rank one constraint for $\mathcal{R}$, Problem 2 can be reformulated as follows:

Problem 3: For the given parameters,

$$\begin{aligned} \min_{\mathcal{R}} \quad & \text{tr}\left(\mathbf{\Phi}_P \mathcal{R}\right) \\ \text{s.t.} \quad & \text{tr}\left([\mathbf{\Phi}_S(k) - \gamma_k \mathbf{\Phi}_N(k)]\mathcal{R}\right) \geq \sigma_d^2 \gamma_k, \quad k \in \mathcal{I}, \\ & \mathcal{R} \succeq 0 \end{aligned} \tag{33}$$

where $\mathbf{\Phi}_P = \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H$, $\mathbf{\Phi}_S(k) = N P_{s,k} \sigma_g^2 \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H$, $\mathbf{\Phi}_N(k) = \sigma_r^2 \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H$.

Note that by relaxing the rank one constraint for $\mathcal{R}$, Problem 2 is converted to Problem 3, which is a semi-definite program (SDP) [16] and it can be solved efficiently by using the standard interior point method for convex optimization [16], [17]. With an additional constraint rank$(\mathcal{R}) = 1$, Problem 3 is equivalent to the original Problem 2. That is, if the optimal solution to Problem 3 has rank one, then it is also the optimal solution to Problem 2. However, there is no guarantee that an algorithm for solving Problem 3 yields the desired rank one solution. In such a case, randomization techniques [18] can be used to obtain a rank-one solution $\mathbf{r}$ from $\mathcal{R}$. However, for this specific problem related to the transparent FF relay design, we provide a stronger result, stated in the following theorem.

Theorem 1: If all the desired SNR constraints except one are satisfied with strict inequality, then the non-trivial optimal solution of Problem 3, which is a relaxed version of Problem 2, always has rank one.

Proof : See the appendix.

Note that the condition in Theorem 1 is mild and is satisfied in many cases. Thus, solving Problem 3 directly yields the solution to the original power minimization problem under subcarrier SNR constraints in many cases.

A.2 Worst subcarrier SNR maximization under a relay transmit power constraint

Now, we consider another useful FF relay design problem for given source power allocation. The second problem is to maximize the SNR of the worst subcarrier channel under a total relay transmit power constraint. This problem is formulated as follows:

Problem 4: For given source power allocation $\{P_{s,k}, k = 0, 1, \cdots, N-1\}$, SR channel $\mathbf{f}$, RD channel static information (σ_g^2, L_g), FF relay filter order L_r, and maximum available relay transmit power $P_{r,\max}$,

$$\max_{\mathbf{r}} \min_{k \in \{0, \cdots, N-1\}} \text{SNR}_k \quad \text{s.t.} \quad P_r \leq P_{r,\max}. \tag{34}$$

By using (30) and (31), Problem 4 can be expressed as

$$\max_{\mathbf{r}} \min_{k\in\{0,\cdots,N-1\}} \frac{NP_{s,k}\sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2} \quad \text{s.t.} \quad \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r} \le P_{r,\max}. \tag{35}$$

By introducing a slack variable τ, the above problem can be rewritten as

$$\begin{aligned} \max_{\mathbf{r}} \quad & \tau \\ \text{s.t.} \quad & \frac{NP_{s,k}\sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2} \ge \tau \ , \qquad k = 0,\ 1,\ \ldots,\ N-1 \\ & \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r} \le P_{r,\max}. \end{aligned} \tag{36}$$

Note that this is a non-convex problem. Again, as in the previous subsection, we convert the problem to a tractable problem by semi-definite relaxation as follows:

Problem 5:

$$\begin{aligned} \max_{\mathcal{R}} \quad & \tau \\ \text{s.t.} \quad & \text{tr}\left(\left(\mathbf{\Phi}_S(k) - \tau \mathbf{\Phi}_N(k)\right)\mathcal{R}\right) \ge \sigma_d^2 \tau, \quad k = 0,\ 1,\ \ldots,\ N-1 \\ & \text{tr}\left(\mathbf{\Phi}_P \mathcal{R}\right) \le P_{r,\max} \\ & \mathcal{R} \succeq 0, \end{aligned} \tag{37}$$

where $\mathbf{\Phi}_S(k)$, $\mathbf{\Phi}_N(k)$, and $\mathbf{\Phi}_P(k)$ are already defined in Problem 3.

In Problem 5, the rank constraint rank$(\mathcal{R}) = 1$ is dropped by semi-definite relaxation as in Problem 3. Note that the relaxed optimization problem is quasi-convex, i.e., for given τ the problem is convex. The solution of the quasi-convex optimization problem can be obtained by solving its corresponding feasibility problem [19]:

$$\begin{aligned} \text{Find} \quad & \mathcal{R} \\ \text{s.t.} \quad & \text{tr}\left(\left(\mathbf{\Phi}_S(k) - \tau \mathbf{\Phi}_N(k)\right)\mathcal{R}\right) \ge \sigma_d^2 \tau, \quad k = 0,\ 1,\ \ldots,\ N-1 \\ & \text{tr}\left(\mathbf{\Phi}_P \mathcal{R}\right) \le P_{r,\max} \\ & \mathcal{R} \succeq 0. \end{aligned} \tag{38}$$

The feasible set in Problem 5 is convex for any value of τ. Let $\tau^\star$ be the optimal value of Problem 5. Then, we can find the solution to Problem 5 by using the fact that the feasibility problem (38) is feasible for $\tau \le \tau^\star$, whereas it is not feasible for $\tau > \tau^\star$. Based on this, we propose a simple bisection algorithm to solve Problem 5 as follows:

Algorithm 1:

Step 1: Choose some appropriate interval s.t. $\tau^\star \in (\tau_L, \tau_R)$.

Step 2: Set $\tau = (\tau_L + \tau_R)/2$.

Step 3: Solve the feasibility problem (38) for τ. If it is feasible, $\tau_L = \tau$. Otherwise, $\tau_R = \tau$.

Step 4: Repeat Steps 1 to 3 until $(\tau_R - \tau_L) < \epsilon$.

Here, ϵ is the allowed error tolerance for τ. Note that the above feasibility problem is a standard SDP problem, which can be solved easily by the interior point method [17]. Due to the relaxation, the matrix $\mathcal{R}$ obtained by solving the relaxed optimization problem may not have rank one in general. In such a case, randomization techniques can be applied to find a rank-one solution.

B. Joint optimization of source power allocation and relay filter

In the previous subsection, we considered the FF relay design under given source power allocation. However, the overall system performance can be improved further by jointly optimizing the source power allocation and the FF relay filter. In this subsection, we consider joint source power allocation and FF relay filter design to maximize the worst subcarrier SNR subject to total source and relay transmit power constraints:

Problem 6: For given SR channel $\mathbf{f}$, RD channel statistic information (σ_g^2, L_g), FF relay filter order L_r, maximum available source transmit power $P_{s,max}$, and maximum available relay transmit power $P_{r,\max}$,

$$\max_{P_{s,0},\cdots,P_{s,N-1},\mathbf{r}} \ \min_{k\in\{0,\cdots,N-1\}} \text{SNR}_k \quad \text{s.t.} \quad \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max} \ \text{ and } \ P_r \le P_{r,\max}. \tag{39}$$

Note that Problem 6 is a complicated non-convex optimization problem, which does not yield an easy solution. To circumvent the difficulty in joint optimization, we approach the problem by using alternating optimization. That is, first the source power allocation is initialized properly and Problem 4 is solved to optimize the relay filter for given source power allocation. (This step can be done by Algorithm 1.) Then, with the given relay filter tap coefficients obtained from Algorithm 1, the source power allocation is optimized, and the alternating procedure is iterated until it converges. Here, the source power allocation problem for a given relay filter is formulated as follows:

Problem 7: For given FF relay filter $\mathbf{r}$, SR channel $\mathbf{f}$, RD channel statistic information (σ_g^2, L_g), maximum allowed source transmit power $P_{s,max}$, and maximum allowed relay transmit power $P_{r,max}$,

$$\max_{P_{s,0},\cdots,P_{s,N-1}} \ \min_{k\in\{0,\cdots,N-1\}} \text{SNR}_k \ \text{ s.t. } \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max}, \ P_r \le P_{r,\max}, \ \text{ and } \text{SNR}_k \ge \tau_0 \ \forall\, k, \tag{40}$$

where τ_0 is the current maximum worst subcarrier SNR value obtained by Algorithm 1.

By introducing a slack variable τ and using (30) and (31), Problem 7 can be rewritten as follows:

$$\max_{P_{s,0},\cdots,P_{s,N-1},\tau} \quad \tau \tag{41}$$

$$\text{s.t.} \quad \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max}, \tag{42}$$

$$\mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r} \le P_{r,\max}, \tag{43}$$

$$\frac{N P_{s,k} \sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2} \ge \tau \ , \qquad k = 0,\ 1,\ \ldots,\ N-1, \tag{44}$$

$$\tau \ge \tau_0. \tag{45}$$

Note that without the relay power constraint (43), the above problem is a simple linear programming (LP) with respect to $P_{s,0}, \cdots, P_{s,N-1}$ and τ. Indeed, the problem is an LP since the relay power constraint can also be written as a linear form in terms of $P_{s,k}$, as shown below. The relay power (31) can be rewritten as

$$\begin{aligned}
\mathbb{E}\{\text{tr}(\mathbf{y}_t \mathbf{y}_t^H)\} &= \text{tr}(\mathbf{R}\mathbf{F}\mathbb{E}\{\tilde{\mathbf{x}}_s \tilde{\mathbf{x}}_s^H\}\mathbf{F}^H \mathbf{R}^H) + \text{tr}(\sigma_r^2 \mathbf{R}\mathbf{R}^H), \\
&= \text{tr}(\mathbb{E}\{\tilde{\mathbf{x}}_s \tilde{\mathbf{x}}_s^H\}\mathbf{F}^H \mathbf{R}^H \mathbf{R}\mathbf{F}) + \text{tr}(\sigma_r^2 \mathbf{R}\mathbf{R}^H), \\
&\overset{(a)}{=} \text{tr}(\tilde{\mathbf{W}}_N \ \mathbb{E}\{\mathbf{s}\mathbf{s}^H\} \ \tilde{\mathbf{W}}_N^H \mathbf{F}^H \mathbf{R}^H \mathbf{R}\mathbf{F}) + \text{tr}(\sigma_r^2 \mathbf{R}\mathbf{R}^H), \\
&\overset{(b)}{=} \sum_{k=0}^{N-1} P_{s,k} \ \text{tr}(\ \mathbf{e}_{k+1} \mathbf{e}_{k+1}^T \ \tilde{\mathbf{W}}_N^H \mathbf{F}^H \mathbf{R}^H \mathbf{R}\mathbf{F} \ \tilde{\mathbf{W}}_N) + \text{tr}(\sigma_r^2 \mathbf{R}\mathbf{R}^H)
\end{aligned} \tag{46}$$

where $\mathbf{e}_k$ is defined in (29), and $\tilde{\mathbf{W}}_N$ is the cyclic prefix extended IDFT matrix given by

$$\tilde{\mathbf{W}}_N = [\mathbf{w}_{N-1},\ \mathbf{w}_{N-2},\ \cdots,\ \mathbf{w}_0,\ \mathbf{w}_{N-1}, \cdots,\ \mathbf{w}_{N-L_g-L_r-L_f+3}]^T.$$

Here, (a) can be verified by using (11) and (b) is due to the assumption of $s[k] \sim \mathcal{CN}(0, P_{s,k})$ for $k = 0, 1, \cdots, N-1$. Using the new expression (46) for the relay transmit power, we obtain an LP optimization problem for the source power allocation from the problem (41-45) as

$$\begin{aligned}
\max_{P_{s,0},\cdots,P_{s,N-1},\tau} \quad & \tau \\
\text{s.t.} \quad & \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max}, \\
& \sum_{k=0}^{N-1} P_{s,k} \ C_1(k) + C_2 \le P_{r,\max} \\
& P_{s,k} \ C_3(k) \ge \tau \ , \qquad k = 0,\ 1,\ \ldots,\ N-1, \\
& \tau \ge \tau_0,
\end{aligned} \tag{47}$$

where $C_1(k) = \text{tr}(\ \mathbf{e}_{k+1}\mathbf{e}_{k+1}^T\ \tilde{\mathbf{W}}_N^H\mathbf{F}^H\mathbf{R}^H\mathbf{R}\mathbf{F}\ \tilde{\mathbf{W}}_N)$, $C_2 = \text{tr}(\sigma_r^2\mathbf{R}\mathbf{R}^H)$, and $C_3(k) = \frac{N\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}$. Now, the proposed alternating optimization for joint source power allocation and relay filter design problem (39) to maximize the worst subcarrier SNR is summarized as follows.

Algorithm 2: Given parameters: $\mathbf{f}$, (σ_g^2, L_g), L_r, $P_{s,\max}$, and $P_{r,\max}$.

Step 1: Initialize $P_{s,k}$ for $k = 0, \cdots, N-1$.

Step 2: Solve Problem 5 by using Algorithm 1. Set τ_0 as $\tau^\star$ obtained from Algorithm 1.

Step 3: For given $\mathbf{r}$ and τ_0 from Step 2, solve Problem (47) to obtain new $P_{s,k},\ k = 0, \cdots, N-1$.

Step 4: Go to Step 2 with τ_L being the solution to Problem (47) in Step 3.

Step 5: Repeat Steps 2 to 4 until $|\tau_0 - \tau_L| < \epsilon$.

Here, ϵ is the allowed error tolerance for τ. The convergence of the proposed iterative algorithm can easily be verified by the monotone convergence theorem applied to $\tau^\star$. Although convergence to the global optimum is not guaranteed, it will be seen in the next section that the proposed joint design approach improves the system performance significantly.

IV. Numerical results

In this section, we provide some numerical results to evaluate the performance of the FF relay design methods proposed in Section III. We considered a relay network with an OFDM transmitter, an FF relay, and a destination node, as described in Section II. The number of OFDM subcarriers was set as $N = 32$ with minimal cyclic prefix covering the overall FIR channel length in each simulation case. The SR channel length and RD channel length were set as $L_f = L_g = 3$, and both SR and RD channel tap coefficients f_l's and g_l's were generated i.i.d. according to a Rayleigh distribution, i.e., $f_l \overset{i.i.d.}{\sim} \mathcal{CN}(0,1)$ for $l = 0, 1, \cdots, L_f - 1$ and $g_l \overset{i.i.d.}{\sim} \mathcal{CN}(0,1)$ for $l = 0, 1, \cdots, L_g - 1$. Throughout the simulation, the relay and the destination had the same noise power $\sigma_r^2 = \sigma_d^2 = 1$ and the source transmit power was 20 dB higher than the noise power. Fig. 3 shows the performance of the first FF relay design method, provided in Problem 3, minimizing the relay transmit power subject to the required SNR constraints on subcarrier channels. We chose $\mathcal{I} = \{0, 1, \cdots, 27\}$ from 32 subcarriers. It is known that for a set of randomly realized propagation channels, it is not easy to always guarantee the desired SNR for each subcarrier channel when the desired SNR value is high [7]. Thus, in Fig. 3, the line was plotted when Problem 3 was feasible for more than 50 % out of 1000 random channel realizations for the given minimum required SNR value for all subcarrier channels in $\mathcal{I}$, and the plotted value is the relay transmit power averaged over feasible channel realizations. It is seen that the required relay transmit power for the same minimum SNR required by the FF relay is significantly reduced when compared with that required by the AF relay. Fig. 4 shows the relay transmit power versus the relay filter length L_r for

various desired minimum SNR values. It is seen that the required relay transmit power for the same desired minimum SNR decreases monotonically with respect to L_r, and the FF relay achieves most of the gain only with a few taps. Next, we evaluated the performance of the second FF relay design method, provided in Algorithm 1, maximizing the worst subcarrier SNR subject to a relay transmit power constraint. Figures 5 and 6 show the worst subcarrier SNR (averaged over 500 channel realizations) versus the FF relay transmitted power for various relay filter lengths and the worst subcarrier SNR (averaged over 500 channel realizations) versus the relay filter length L_r for various relay transmit power values, respectively. As in the previous case of relay transmit power minimization, the gain by the FF relay over the AF relay is significant and most of the gain is achieved by only a few filter taps for the FF relay. Recall that our FF relay design is based on the channel statistic information for the RD channel. Even exploiting the channel statistic (not the channel state) for the RD channel yields significant gain over the AF relay. Now, let us examine the robustness of the proposed FF relay design, Algorithm 1, against channel static mismatch. Still, the i.i.d. RD channel model with $L_g = 3$ and $\sigma_g^2 = 1$ for all taps was used for Algorithm 1 to obtain the relay filter, but the true channel was generated randomly according to a different channel statistic, i.e., a different L_g and/or different channel power profile. Fig. 7 shows the worst subcarrier SNR (averaged 500 over channel realizations) versus the relay transmit power in the case of channel static information mismatch. Here, the worst subcarrier SNR in Fig. 7 was computed based on the true channel state and the relay filter obtained by Algorithm 1 with wrong channel static information. It is seen in the figure that the proposed FF relay design method is robust against the channel static mismatch, and the proposed FF relay outperforms the AF relay even in this case. Finally, we investigated the performance of the joint source power allocation and FF relay filter design to maximize the worst subcarrier SNR, provided in Algorithm 2, and the result is shown in Fig. 8. It is seen that the joint optimization method significantly outperforms the optimization of only the relay filter presented in Algorithm 1.

V. Conclusion and Discussion

In this paper, we have considered the FF relay design for OFDM systems for transparent relay operation under two design criteria of minimizing the relay transmit power subject to per-subcarrier SNR constraints and maximizing the worst subcarrier SNR subject to a total relay transmit power constraint. We have shown that the two design problems reduce to tractable SDP problems and that the FF relay designed by solving the obtained SDP problems significantly outperforms the AF relay under both criteria. In closing, we discuss several issues and the direction of our future research. First, we assumed channel state information for the SR channel and channel statistic information for the RD channel. Although this assumption originates

from the consideration of cheap transparent relay operation for FF relays, the assumption of channel statistic information for the RD channel makes the proposed FF relay design method useful even for broadcasting purposes when the destination nodes in the relay cell have similar channel statistics. Second, we assumed SISO OFDM systems. However, most current OFDM systems employ MIMO communications, and extension to the MIMO case is left as a future work. Third, FF relay design for rate maximization is also an interesting problem. This problem can be handled by a similar approach to that used for the rate maximization in single carrier systems in [10]. See the end of the Appendix for this issue.

APPENDIX

Proof of Theorem 1

By introducing a slack variable τ, we convert Problem 3 to the following equivalent problem:

$$\min_{\tau,\mathcal{R}} \quad \tau \tag{48}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{\Phi}_P\mathcal{R}) \le \tau, \tag{49}$$

$$\mathrm{tr}([\mathbf{\Phi}_S(k) - \gamma_k\mathbf{\Phi}_N(k)]\mathcal{R}) \ge \sigma_d^2\gamma_k, \quad k = 0, 1, \dots, N-1, \tag{50}$$

$$\tau \ge 0, \tag{51}$$

$$\mathcal{R} \succeq 0. \tag{52}$$

The Lagrange dual function for the above problem is given by

$$g(\lambda, \{\mu_k\}, \nu, \mathbf{\Psi}) = \inf_{\tau,\mathcal{R}} \left((1-\lambda-\nu)\tau + \sum_{k=0}^{N-1} \mu_k\gamma_k\sigma_d^2 + \mathrm{tr}(\{ \underbrace{\lambda\mathbf{\Phi}_P - \sum_{k=0}^{N-1} \mu_k[\mathbf{\Phi}_S(k) - \gamma_k\mathbf{\Phi}_N(k)]}_{=:\mathbf{Q}(\lambda,\{\mu_k\})} - \mathbf{\Psi}\}\mathcal{R}) \right), \tag{53}$$

where $\lambda \ge 0$, $\{\mu_k \ge 0\}$, $\nu \ge 0$ and $\mathbf{\Psi} \succeq 0$ are the dual variables associated with (49), (50), (51), and (52), respectively. If $1-\lambda-\nu \ne 0$ or $\mathbf{Q}(\lambda, \{\mu_k\}) - \mathbf{\Psi} \ne 0$, then the dual function value is minus infinity or we have trivial solutions $\tau = 0$ and/or $\mathcal{R} = \mathbf{0}$. Thus, for the nontrivial feasibility of τ and $\mathcal{R}$, we have $1 - \lambda = \nu$ (≥ 0) and $\mathbf{Q}(\lambda, \{\mu_k\}) = \mathbf{\Psi}$ $(\succeq 0)$. Then, the Lagrange dual function is easily obtained as $g(\lambda, \{\mu_k\}, \nu, \mathbf{\Psi}) = \sum_{k=0}^{N-1} \mu_k\gamma_k\sigma_d^2$ and the corresponding dual problem is given by

$$\begin{aligned} \max_{\lambda,\{\mu_k\}} \quad & \sum_{k=0}^{N-1} \mu_k\gamma_k\sigma_d^2 \\ \text{s.t.} \quad & 0 \le \lambda \le 1, \ \ \mathbf{Q}(\lambda, \{\mu_k\}) \succeq 0, \ \ \mu_k \ge 0, \ \ k = 0,\ 1,\ \dots,\ N-1. \end{aligned} \tag{54}$$

Let $\lambda^\star,\ \{\mu_k^\star\},\ \tau^\star,\ \mathbf{Q}^\star$ and $\mathcal{R}^\star$ be the optimal values for the problem. ($\nu^\star$ and $\mathbf{\Psi}^\star$ are automatically determined based on these quantities. The dependence of $\mathbf{Q}$ on λ and $\{\mu_k\}$ is not shown explicitly for notational

simplicity from here on.) From the complementary slackness conditions for (49) and (50), we have

$$\lambda^\star \left(\text{tr}\left(\mathbf{\Phi}_P \mathcal{R}^\star\right) - \tau^\star\right) + \sum_{k=0}^{N-1} \mu_k^\star \left(\gamma_k \sigma_d^2 - \text{tr}\left([\mathbf{\Phi}_S(n) - \gamma_k \mathbf{\Phi}_N(k)]\mathcal{R}^*\right)\right) = 0, \tag{55}$$

which is equivalent to

$$\left(\sum_{k=0}^{N-1} \mu_k^\star \gamma_k \sigma_d^2 - \lambda^\star \tau^\star\right) + \text{tr}\left(\underbrace{\{\lambda^\star \mathbf{\Phi}_P - \sum_{k=0}^{N-1} \mu_k^\star [\mathbf{\Phi}_S(k) - \gamma_k \mathbf{\Phi}_N(k)]\}}_{=\mathbf{Q}^\star} \mathcal{R}^\star\right) = 0. \tag{56}$$

Since the problem (48-52) is a convex optimization problem, the duality gap is zero, i.e., $\sum_{k=0}^{N-1} \mu_k^\star \gamma_k \sigma_d^2 = \tau^\star$. Thus, both the first and second terms in the left-hand side (LHS) of (56) are nonnegative since $\sum_{k=0}^{N-1} \mu_k^\star \gamma_k \sigma_d^2 - \lambda^\star \tau^\star = \tau^\star(1-\lambda^\star) \geq 0$ and tr$(\mathbf{Q}^\star \mathcal{R}^\star) \geq 0$. (The trace of the product of two positive semi-definite matrices is nonnegative [20].) Therefore, $\lambda^\star = 1$ and tr$(\mathbf{Q}^\star \mathcal{R}^\star) = 0$. It is obvious that $\mathbf{Q}^\star \not\succ 0$ for a nontrivial $\mathcal{R}^\star$ from tr$(\mathbf{Q}^\star \mathcal{R}^\star) = 0$, i.e., the $L_r \times L_r$ matrix $\mathbf{Q}^\star$ does not have full rank. This is because tr$(\mathbf{Q}^\star \mathcal{R}^\star) = \sum_i \sigma_i \text{tr}(\mathbf{Q}^\star \mathbf{u}_i \mathbf{u}_i^H) = \sum_i \sigma_i (\mathbf{u}_i^H \mathbf{Q}^\star \mathbf{u}_i)$, where $\mathcal{R} = \sum_i \sigma_i \mathbf{u}_i \mathbf{u}_i^H$ is the eigen-decomposition of $\mathcal{R}^\star$. (If $\mathbf{Q}^\star \succ 0$, then tr$(\mathbf{Q}^\star \mathcal{R}^\star) > 0$.) Note from (56) that

$$\mathbf{Q}^\star = \mathbf{\Phi}_P - \sum_{k=0}^{N-1} \mu_k^\star \left(\mathbf{\Phi}_S(k) - \gamma_k \mathbf{\Phi}_N(k)\right), \tag{57}$$

where $\mathbf{\Phi}_P$ is a positive definite matrix defined in (31), $\mathbf{\Phi}_N(k)$ is a positive semi-definite matrix defined in (28), and $\mathbf{\Phi}_S(k)$ defined in (26) is a rank-one matrix by Lemma 2. Now, under the assumption that all the SNR constraints except one are satisfied with strict inequality, we have $\mu_i \neq 0$ for some i and $\mu_j = 0, \forall j \neq i$ from the complementary slackness conditions. In this case, $\mathbf{Q}^\star$ is given by

$$\mathbf{Q}^\star = \underbrace{\mathbf{\Phi}_P + \mu_i^\star \gamma_i \mathbf{\Phi}_N(i)}_{\text{rank } L_r} - \underbrace{\mu_i^\star \mathbf{\Phi}_S(i)}_{\text{rank } 1}. \tag{58}$$

Due to the structure of $\mathbf{Q}^\star$ in (58), the rank of $\mathbf{Q}^\star$ is larger than or equal to $L_r - 1$. Since $\mathbf{Q}^\star \not\succ 0$, rank $(\mathbf{Q}^\star) = L_r - 1$. Since $0 = \text{tr}(\mathbf{Q}^\star \mathcal{R}^\star) = \sum_{i=1}^{L_r-1} \eta_i (\mathbf{v}_i^H \mathcal{R}^\star \mathbf{v}_i)$ (where $\mathbf{Q}^\star = \sum_{i=1}^{L_r-1} \eta_i \mathbf{v}_i \mathbf{v}_i^H$ is the eigen-decomposition of $\mathbf{Q}^\star$), we conclude that $\mathcal{R}^\star$ has nullity $L_r - 1$ and thus has rank one under the assumption of Theorem 1. ■

Lemma 2: If $N - L_f + 1 > L_f + L_g - 1$, $\mathbf{\Phi}_S(k)$ has rank one regardless of the value of k.

Proof of Lemma 2: Recall that (see (26) and (33))

$$(NP_{s,k}\sigma_g^2)^{-1} \mathbf{\Phi}_S(k) = \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H, \quad \tilde{\mathbf{K}}_k = \tilde{\mathbf{I}}_{L_g} \otimes \mathbf{K}_k, \quad \mathbf{K}_k = \mathbf{F}^* \mathbf{T}^* \mathbf{w}_k \mathbf{w}_k^H \mathbf{T}^T \mathbf{F}^T. \tag{59}$$

Let $\mathbf{E}_1$ in (27) be partitioned as $\mathbf{E}_1 = \left[\mathbf{E}_1^{(1)},\ \mathbf{E}_1^{(2)},\ \ldots,\ \mathbf{E}_1^{(N+L_g-1)}\right]$. Note that $\mathbf{F}$ and $\mathbf{T}$ are Toeplitz matrices and that $\mathbf{w}_k\mathbf{w}_k^H$ is also a Toeplitz matrix regardless of k due to the property of DFT matrices. It is not difficult to show that $\mathbf{K}_k(1:N-L_f+1, 1:N-L_f+1)$ is a Toeplitz matrix, where $\mathbf{A}(a:b, c:d)$ denotes a submatrix of $\mathbf{A}$ composed of the rows from a to b and columns from c to d. Now, $\mathbf{\Phi}_S(k)$ can be rewritten as

$$(NP_{s,k}\sigma_g^2)^{-1}\mathbf{\Phi}_S(k) = \mathbf{E}_1^{(1)}\mathbf{K}_k\mathbf{E}_1^{(1)H} + \mathbf{E}_1^{(2)}\mathbf{K}_k\mathbf{E}_1^{(2)H} + \cdots + \mathbf{E}_1^{(L_g)}\mathbf{K}_k\mathbf{E}_1^{(L_g)H}. \tag{60}$$

Here, the operation $\mathbf{E}_1^{(i)}\mathbf{K}_k\mathbf{E}_1^{(i)H}$ extracts a $L_r \times L_r$ submatrix $\mathbf{K}_k(i:L_r+i-1,\ i:L_r+i-1)$ from $\mathbf{K}_k$. If $N-L_r+1 > L_r+L_g-1$, this operation extracts the same submatrix from $\mathbf{K}_k$ regardless of i since $\mathbf{K}_k(1:N-L_r+1, 1:N-L_r+1)$ is a Toeplitz matrix. Thus, we have

$$(NP_{s,k}\sigma_g^2)^{-1}\mathbf{\Phi}_S(k) = L_g\mathbf{E}_1^{(1)}\mathbf{K}_k\mathbf{E}_1^{(1)H} = L_g(\mathbf{E}_1^{(1)}\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k)(\mathbf{E}_1^{(1)}\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k)^H, \tag{61}$$

and $\mathbf{\Phi}_S(k)$ has rank one if the condition $N-L_f+1 > L_t+L_g-1$. ■

Joint Optimization of Source Power Allocation and FF Relay Filter for Rate Maximization

This problem can be formulated by using the results in this paper, and the problem is expressed as follows:

Problem 8: For given $\mathbf{f}$, (L_g, σ_g^2), L_r, $P_{s,max}$ and $P_{r,max}$,

$$\max_{P_{s,0},\cdots,P_{s,N-1},\ \mathbf{r}} \quad \sum_{k=0}^{N-1}\frac{1}{2}\log\left(1+\frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}\right) \tag{62}$$

$$\text{s.t.} \quad \sum_{k=0}^{N-1}P_{s,k} \le P_{s,\max}, \tag{63}$$

$$\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \le P_{r,\max}. \tag{64}$$

The problem (62-64) is not a convex problem and thus, does not yield a closed-form solution. However, this problem is a discrete frequency version of the problem considered in [10]. Thus, the approach in [10] based on the projected gradient method can be applied with the new design variable $\mathbf{u} := [P_{s,0},\cdots,P_{s,N-1}, r_0, r_1,\cdots,r_{L_r-1}]^T$ and $\phi(\mathbf{u}) := \sum_{k=0}^{N-1}\frac{1}{2}\log\left(1+\frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}\right)$. This approach yields a nontrivial rate gain over the AF scheme. Please see [10] and [21] for detail.

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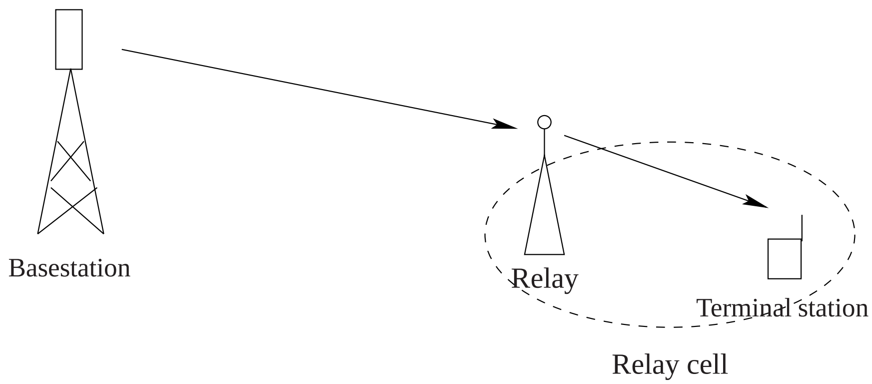

Fig. 1

THE CONSIDERED RELAY NETWORK

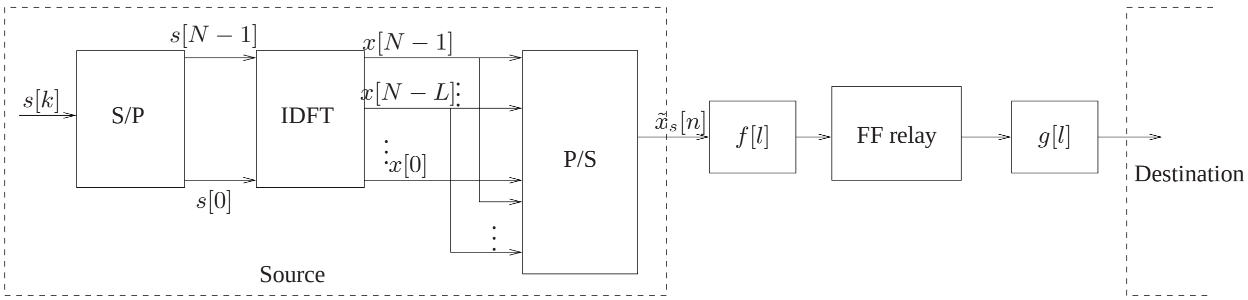

Fig. 2

THE CONSIDERED RELAY NETWORK WITH AN OFDM TRANSMITTER, AN FF RELAY, AND A DESTINATION NODE

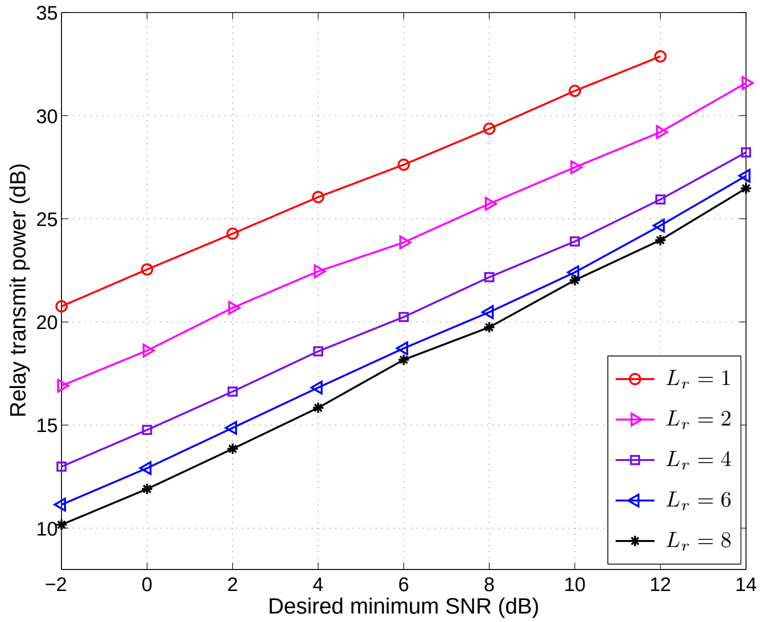

Fig. 3

FF RELAY TRANSMIT POWER VERSUS DESIRED MINIMUM SNR.

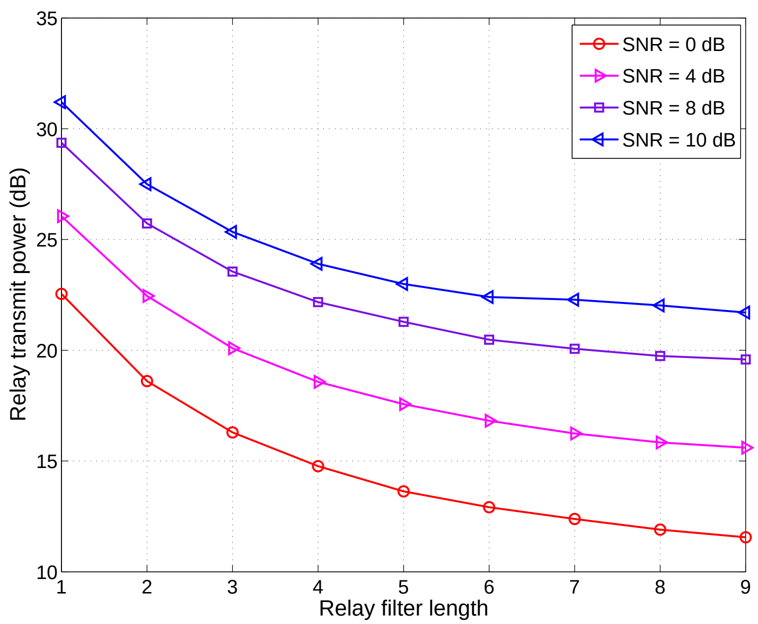

Fig. 4

FF RELAY TRANSMIT POWER VERSUS FF RELAY FILTER LENGTH L_r.

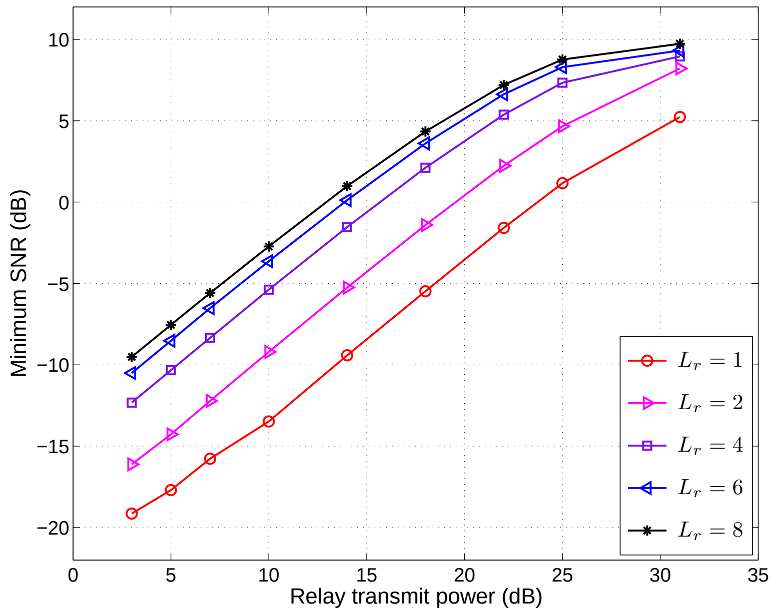

Fig. 5

THE WORST SUBCARRIER SNR VERSUS RELAY TRANSMIT POWER.

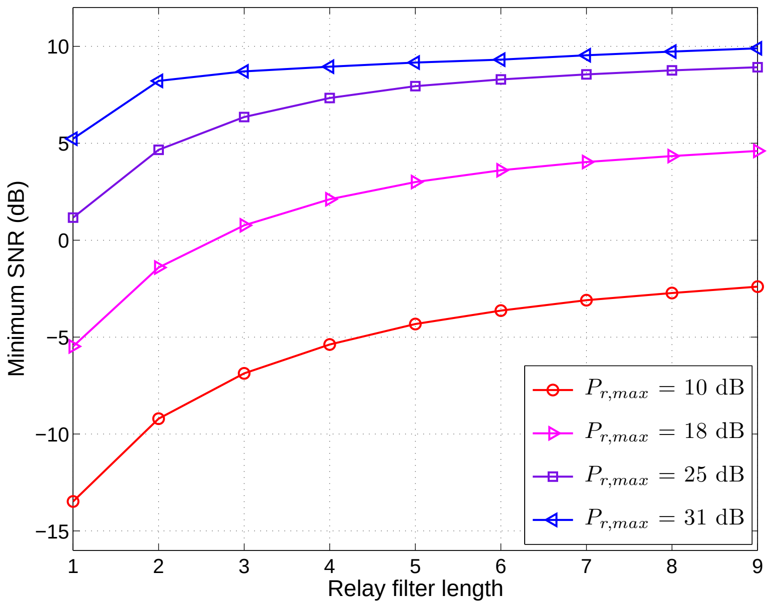

Fig. 6

THE WORST SUBCARRIER SNR VERSUS FF RELAY FILTER LENGTH L_r.

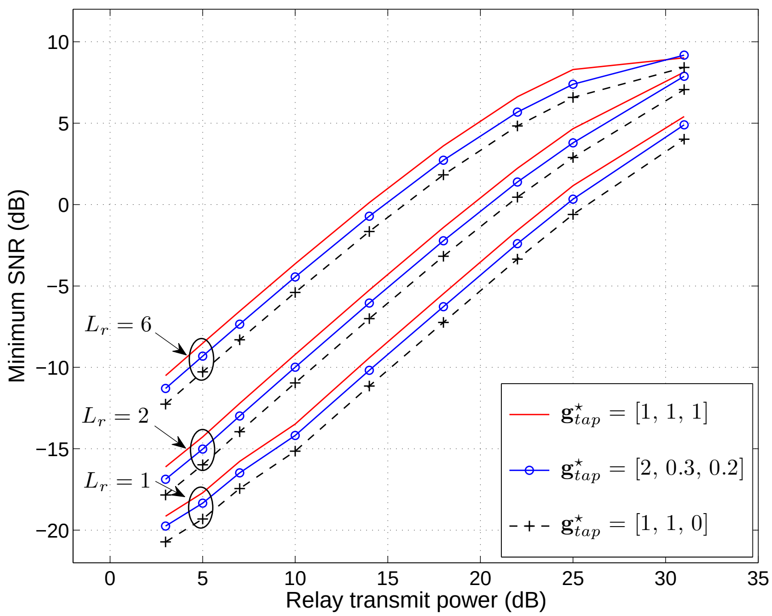

Fig. 7

THE WORST SUBCARRIER SNR VERSUS FF RELAY TRANSMIT POWER: MISMATCHED CHANNEL STATISTIC INFORMATION

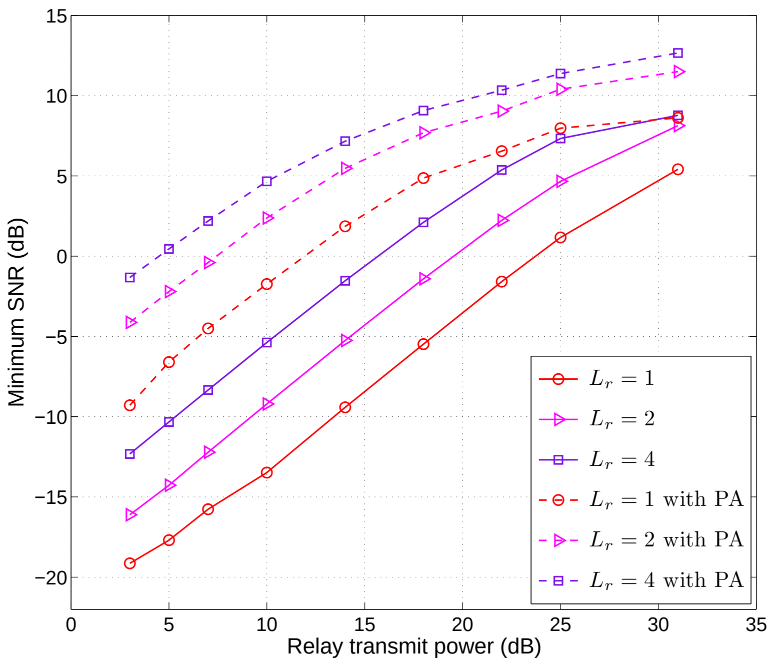

Fig. 8

PERFORMANCE OF JOINT OPTIMIZATION FOR THE WORST SUBCARRIER SNR